Astro 201; Project Set #5

due 4/13/2012

Problems on non-LTE ionization and excitation, and 21 cm radiation

The Rise of Stromgren Spheres

Consider an O-star that turns on and begins radiating a surrounding medium of neutral hydrogen. This produces an ionized region, initially small but growing in size until the total number of ionizations balance recombinations – i.e., when the radius of the HII region reaches the Stromgren radius, R_s. Here we derive a simple time dependent solution for the growth of an HII region.[1]

We will imagine the HII region to have a relatively sharp outer boundary at radius $R(t)$, which marks the ionization front. Defining $\dot{N}_{\rm ion}$ ($\dot{N}_{\rm rec}$) as the total number of photoionizations (recombinations) per second that occur in the surrounding gas, we have the expression

$$\frac{dN_{\rm tot}}{dt} = \dot{N}_{\rm ion} - \dot{N}_{\rm rec} \qquad (1)$$

where $N_{\rm tot}(t)$ is the total number of hydrogen ions contained within the radius $R(t)$. When the rate of ionizations exceeds the rate of recombinations, the number of ionized atoms in the HII region (and hence its radius) will increase with time[2].

For simplicity, we'll assume that the O-star has luminosity L and emits all photons at the ionization threshold – i.e., at a single wavelength $\lambda_0 = 912$ Å. We'll assume the surrounding gas is pure hydrogen with a number density, n, that is constant with radius.

a) Insert expressions for, $\dot{N}_{\rm ion}$, $\dot{N}_{\rm rec}$ in equation 1 and derive an analytic expression for $R(t)$. What is the final radius of the HII region?

b) What is the timescale for the HII region to grow to its final radius? If the gas has a density $n = 1$ cm^{-3} and a temperature $T = 10^4$ K, does the HII region have time to reach its final radius before the O-star dies?

c) Write down the expression for the velocity of the outer edge (ionization front) of the HII region. If the O-star has a luminosity $L = 10^{39}$ ergs s^{-1}, how does the typical velocity of the HII expansion compare to the sound speed for the above temperature and density?[3]

So far, our arguments have all concerned the global ionization state. Let's consider in more detail the the radial ionization structure inside the HII region once it has reached its final (equilibrium) radius.

[1] We consider here the growth only due to photoionization. Because the surrounding gas is being radiatively heated at the same time, its pressure increases and the ionized region may expand hydrodynamically as well. We will see, however, that this hydrodynamical expansion should occur mainly after we have reached global photoionization balance.

[2] Setting $dN_{\rm tot}/dt = 0$ simply reproduces the Stromgren argument for the size of an HII region in equilibrium.

[3] Supersonic expansion of the ionization front suggests we do not need to consider the hydrodynamical effects just yet. See Chaper 20 of Shu's hydrodynamics book for a detailed description of the various interesting dynamical effects in HII regions.

Although the gas within an HII region is highly ionized, some small fraction of neutral hydrogen remains.

To make the problem more tractable, we'll assume that the radius of the O-star is very small and that there is no appreciable attenuation of the radiation field – i.e., $J_\nu = L_\nu / 4\pi r^2$. This should be OK at least in the inner parts of the HII region. Ignore any re-emission or scattering of ionizing photons in the HII region.

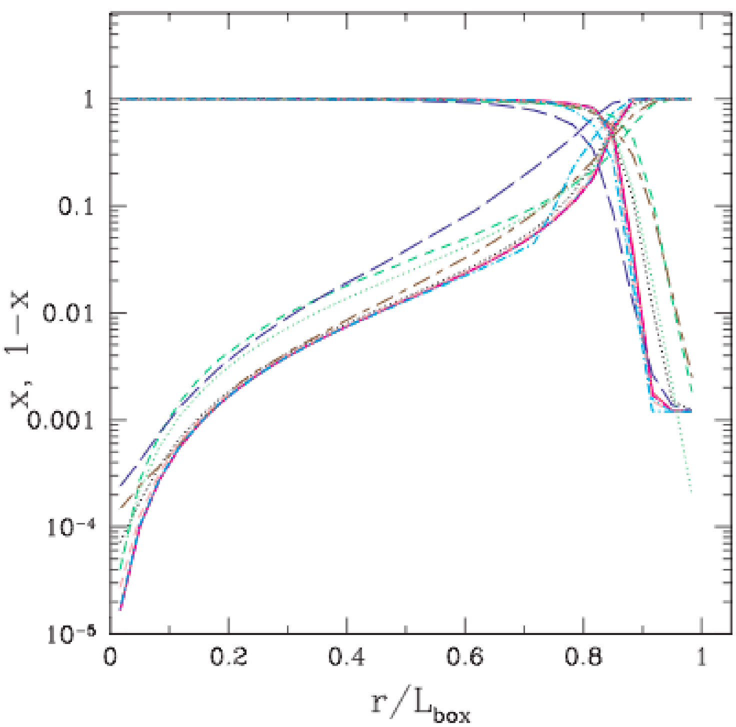

Figure 1: The fraction of neutral hydrogen (lower lines) and ionized hydrogen (upper lines) as a function of radius for a HII region calculation. The different lines compare the results from different numerical codes (deviations are due to the fact that different codes use different approximations to solve the radiative transfer equation and different values for the recombination coefficient and cross-section). From Iliev et al., 2000

d) Assuming photoionization equilibrium holds at each radius r, Derive an expression for the fraction of neutral hydrogen $x_{\rm HI}(r) = n_{\rm HI}/n_{\rm H}$ as a function for radius. Check that you get the right limits at some finite r when $L \to 0$ and $L \to \infty$.

e) Show that in the limit of small radii ($r \ll R_s$) there is a simple expression for the neutral fraction which gives the scaling $x_{\rm HI} \propto nL^{-1}r^2$

Comment: Figure 1 shows the ionization structure for a similar setup, calculated using full radiation transport codes, which should be qualitatively similar to your analytic result, at least for the inner regions. At the edge of the Strogmren sphere, where the matter begins to go neutral and the optical depth begins to get large, there is a very sharp transition to neutral gas.

Flipping Spins at the Epoch of Reionization

OBSERVATIONS OF THE 21 CM LINE at redshifts of $z \geq 10$ could probe the distribution of neutral hydrogen at the epoch of reionization, providing new insight into astrophysics and cosmology. This is the goal of a new generation of radio interferometers e.g., the Murchison Widefield Array (MWA), the LOw Frequency ARray (LOFAR), the Precision Array to Probe the Epoch of Reionization (PAPER), the 21 cm Array (21CMA), and the Giant Meterwave Radio Telescope (GMRT), along with next generation instruments like the Square Kilometer Array (SKA).

Due to cosmological redshift, the 21 cm signal at different epochs will map to different wavelengths; hence we may be able to read off the evolution of hydrogen gas from a spectrum. Figure 2, from the nice review article of Prichard and Loeb (2012) , shows what the spectrum might look like. At different redshifts, the 21cm line may be seen in either emission or absorption relative to the background radiation source, which is generally the CMB. Here we consider the basic physics of the 21 cm fluctuations, and familiarize ourselves with the terminology used in the literature. In particular, the 21 cm community is fond of defining a great variety of "temperatures".

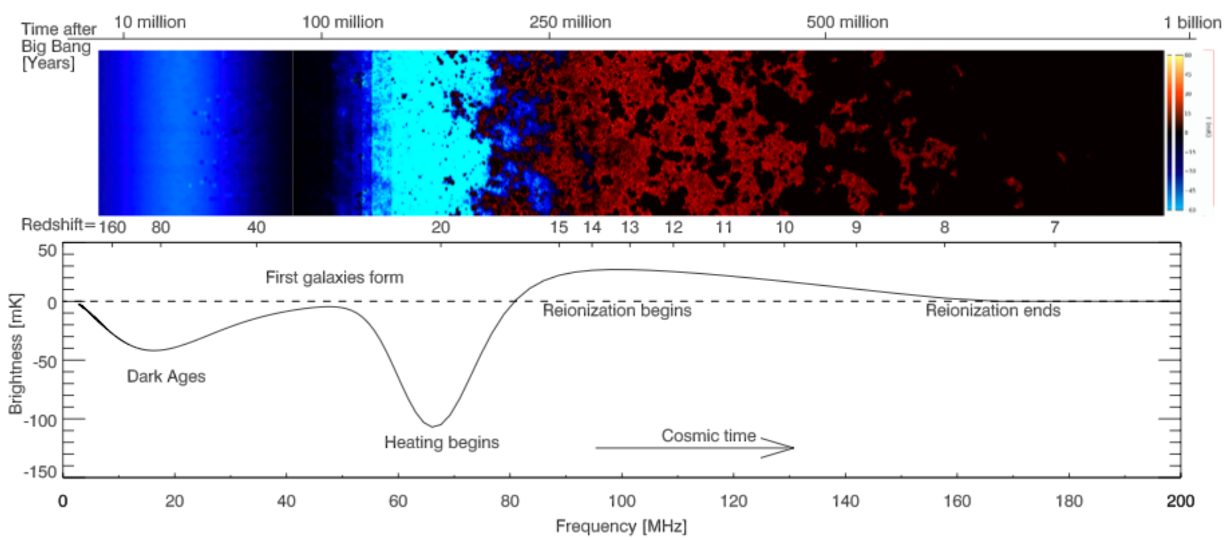

Figure 2: Schematic picture of the predicted cosmological 21 cm signature through the epoch of reionizaiton. From Prichard and Loeb (2012) .

First, define the constant $T_\star = h\nu_{\rm fs}/k = 0.068$ K, where $\nu_{\rm fs}$ is the frequency of the 21 cm line. We will be in the regime $T \gg T_\star$ for any reasonable temperature we may encounter, and are thus in the Raleigh-Jeans limit. In this case, people describe the observed specific intensity using the brightness temperature, T_b, defined by

$$I_\nu(\nu_{\rm fs}) = \frac{2\nu_{\rm fs}^2}{c^2} k T_b \tag{2}$$

The value T_b may or may not have anything to do with the actual kinetic temperature, T_K of the gas being observed. As defined, it is merely an alternative way of expressing I_ν.

We define another temperature, the spin temperature, T_s, which describes the hyperfine level populations[4]

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-h\nu_{\rm fs}/kT_s} \tag{3}$$

When the gas is in LTE we have the identification $T_s = T_K$; i.e., the spin temperature is the same as the actual gas temperature. If LTE does not hold, T_s does not correspond to any real thermodynamic temperature, and is simply a convenient parameter to describe the ratio n_1/n_0.

The hyperfine level populations will be influenced by radiative transitions. We define the radiation temperature, T_γ, in terms of the local mean intensity of the radiation field, J_ν

$$J_\nu(\nu_{\rm fs}) = B_\nu(T_\gamma, \nu_{\rm fs}) \tag{4}$$

This definition does not necessarily assume the radiation field is a blackbody – we are simply defining T_γ as the temperature for which the Planck function equals J_ν at the frequency $\nu_{\rm fs}$. In our case, the radiation field is due (primarily) to the CMB, which actually is a blackbody with $T_\gamma = 2.7(1+z)$.

a) Consider a specific intensity beam from the background CMB passing through a hydrogen cloud of optical depth τ. Show that the observed fluctuation in the brightness temperature of the 21 cm line (relative to the background CMB brightness temperature) is given by[5]

$$\delta T_b = (T_s - T_\gamma)(1 - e^{-\tau}) \tag{5}$$

If we can determine T_s, we can predict whether the 21 cm line should be seen in emission ($\delta T_b > 0$) or absorption ($\delta T_b < 0$). Caclulating the spin temperature, however, is difficult because it is affected by various astrophysical processes.

b). The hyperfine levels will generally be in statistical equilibrium (not necessarily LTE) in which the spin flip transitions are in steady state. Assume that transition between the levels are due to either collisions (e.g., impacts with other hydrogen atoms) or radiation. Show using the Einstein coefficients that the expression for the spin temperature[6] is

[4] To be consistent with Prichard and Loeb, I'll use a subscript 1 to describe the excited (F=1) hyperfine state of hydrogen and subscript 0 for the ground state ($F = 0$). For other transitions, the spin temperature is usually called the excitation temperature.

[5] We won't gone into the effects of cosmological redshift, which reduce the overall intensity of the fluctuation. If included, the right hand side of this expression should be divided by a factor of $(1+z)$.

[6] Recall $T \gg T_\star$ for any T.

$$T_s^{-1} = \frac{T_\gamma^{-1} + x_c T_K^{-1}}{1 + x_c} \tag{6}$$

where

$$x_c = \frac{C_{10}}{A_{10}} \frac{T_\star}{T_\gamma} \tag{7}$$

Determine the value of T_s in the two limits $x_c \ll 1$ and $x_c \gg 1$ and briefly explain why these limits makes sense.

c) There is a critical density[7] where $x_c \sim 1$. As a rough estimate of its value, consider the case where C_{10} is due to neutral hydrogen atoms colliding with each other, and assume the cross-section for collisional de-excitation is just the geometrical cross section of an H atom. Determine the critical hydrogen density for the specific conditions $T_\gamma = 2.7$ K and $T_K = 100$ K.

[7] This is a slightly different critical density that we defined in class, in which we only looked at the ratio of collisional to spontaneous de-excitation C_{10}/A_{10}. We see that taking into account the transitions driven by the incident radiation field introduces the additional factor $T_\star/T_\gamma$.

There is another important effect that can ”flip the spin" of a hydrogen atom – the scattering of lyman alpha (L_α) photons[8]. A L_α photon can excite a hydrogen atom in either of the two $n = 1$ hyperfine states to an $n = 2$ level. The subsequent emission of a L_α photon can return the electron to either of the two $n = 1$ hyperfine states, effectively causing a transition between these levels. This is known as the Wouthuysen-Field effect.

[8] Lyman alpha photons will be produced in abundance once stars/AGN form and produce HII regions.

The L_α line is so optically thick that we expect the radiation field near line center to be coupled to the gas temperature and reach its LTE value[9]

$$J_\nu(\nu_{L\alpha}) = B_\nu(T_K, \nu_{L\alpha}) \tag{8}$$

[9] The coupling of the L_α photons and the gas thermal energy pool is subtle, but related to the energy exchange that occurs when an atom recoils upon scattering a L_α photon. Though the energy exchange of any one scattering is small, many repeated scatterings can effectively couple the L_α radiation to the gas temperature.

Let P_{10} be the rate at which L_α photons drive transitions from the ground to the excited hyperfine level. From LTE arguments we would also then expect that the total rate of transitions from 0 to 1 is

$$P_{01} = P_{10} \frac{g_1}{g_0} e^{-h\nu_{\mathrm{fs}}/kT_K} \tag{9}$$

d) Show that a more general expression for the spin temperature

which includes the Wouthuysen-Field effect (and assumes the L_α radiation field is well-coupled to the gas temperature) is

$$T_s^{-1} = \frac{T_\gamma^{-1} + (x_c + x_\alpha) T_K^{-1}}{1 + x_c + x_\alpha} \tag{10}$$

and write down the expression for x_α in terms of P_{10}.

e) Now we can make qualitative predictions of the kind of signal we expect from cosmological 21 cm measurements. For each epoch below, give a very brief explanation as to why you predict we should see the 21 cm fluctuations (relative to the CMB) in emission, in absorption, or not at all.

1. ($200 \leq z \leq 1100$): After recombination at $z \approx 1100$, the gas density is high enough that the gas and CMB radiation remain thermally coupled[10] and have the same temperature. There are as yet no sources of lyman alpha or ionizing photons.

2. ($40 \leq z \leq 200$): As cosmological expansion continues, the gas and radiation go out of equilibrium and their temperatures evolve independently and adiabatically. The gas density is still well above the critical density defined in **c)**, though.

3. ($30 \leq z \leq 40$): The gas density drops below the critical density such that radiative transitions from the CMB set the level populations.

4. ($15 \leq z \leq 30$): The first sources (stars, AGN) turn on, and produce enough L_α photons that the hyperfine level populations are set by the Wouthuysen-Field effect. The gas is still cool from adiabatic expansion, so $T_K < T_\gamma$.

5. ($7 \leq z \leq 15$): The radiation (mostly x-rays) from sources heat the gas to the point that T_K becomes larger than the CMB temperature. Lyman alpha coupling is still effective (i.e., $x_\alpha \gg 1$).

6. ($z \leq 7$): Enough ionizing radiation from the sources has been emitted that reionization is complete – i.e., essentially all of the neutral hydrogen in the intergalactic medium has been ionized.

[10] This is typically mediated by the energy exchange from compton scattering

Comment: We can now better understand the predicted signal drawn in Figure 2. The above run down is of course just a guess at what the 21 signature should look like, and at what redshifts we might expect transitions. We still have a lot to learn about cosmological reionization and the sources of radiation that influence the intergalactic medium. Having actual 21 cm observations at these epochs obviously would teach us a lot.